

Faraday's Law of Electromagnetic Induction!

Faraday in 1831 observed experimentally that whenever magnetic flux linked with a closed circuit changes, an electromotive force (e.m.f) is induced in the same. The e.m.f so developed is called induced e.m.f and the resulting current in the circuit is known as the induced current. The phenomenon is called electromagnetic induction.

The results of Faraday's experiment led to the development of the following two laws of the electromagnetic induction.

1. Neumann's law: The induced e.m.f. in a circuit is equal to the time rate of change of the magnetic flux linked with the circuit.

If ϕ be the flux linked with the circuit at any time t , then $\frac{d\phi}{dt}$ gives the time rate of change of flux. According to this law, the magnitude of induced e.m.f

$$|e| = \frac{d\phi}{dt}$$

2. Lenz's law: The direction of induced e.m.f or the current is such that it will oppose the very cause for which that is due (i.e. change of flux in the circuit).

$$e = -\frac{d\phi}{dt}$$

The negative sign signifies that e opposes the change of flux. That is why induced e.m.f is sometimes referred as back e.m.f. If R be the resistance of the closed circuit, then induced current -

$$i = \frac{e}{R} = -\frac{1}{R} \frac{d\phi}{dt}$$

Integral form of Faraday's law of electromagnetic induction:

If ϕ be the magnetic flux with the circuit at any time t , then the induced e.m.f is given by

$$\mathcal{E} = - \frac{d\phi}{dt} \quad \text{--- (1)}$$

If \vec{E} be the electric field in space, then by definition, the e.m.f. around a closed path C is

$$\mathcal{E} = \oint_C \vec{E} \cdot d\vec{l}$$

$$\therefore \oint_C \vec{E} \cdot d\vec{l} = - \frac{d\phi}{dt} \quad \text{--- (2)}$$

If \vec{B} be the magnetic induction vector and $d\vec{s}$ an element of surface, then

$$\phi = \int_S \vec{B} \cdot d\vec{s}$$

$$\therefore \oint_C \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \int_S \vec{B} \cdot d\vec{s}$$

$$\therefore \boxed{\oint_C \vec{E} \cdot d\vec{l} = - \int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}} \quad \text{--- (3)}$$

Differential form of Faraday's law of electromagnetic induction.

By Stoke's law

$$\oint_C \vec{E} \cdot d\vec{l} = \int_S (\nabla \times \vec{E}) \cdot d\vec{s}$$

Using eqⁿ (3)

$$\int_S (\nabla \times \vec{E}) \cdot d\vec{s} = - \int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

Since S is arbitrary

$$\therefore \boxed{\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}}$$